

Outline



- Introduction
- Fundamental Mathematical Operators
 - Divergence and Vorticity
- Vector Glyphs
- Vector Color Coding
- Color coding on 2D surfaces

Introduction

- A vector is a tuple of n scalar components

$$v = (v_1, \dots, v_n), v_i \in \mathbb{R}.$$

- An n -dimensional vector describes a position, direction, rate of change, or force in \mathbb{R}^n .
- Majority of visualization applications deal with data that describes physical phenomena in 2 or 3-dimensional space.
- As a consequence, most visualization software defines all vectors to have three components.
- 2D vectors are modeled as 3D vectors with the third (z) component equal to null.

Fundamental mathematical operators



- Fundamental mathematical operators that are used to analyze vector fields.
- Divergence and vorticity are important quantities for vector field visualization, but also for the visualization and processing of other types of datasets, such as meshes, images, and scalar and tensor fields.

Divergence

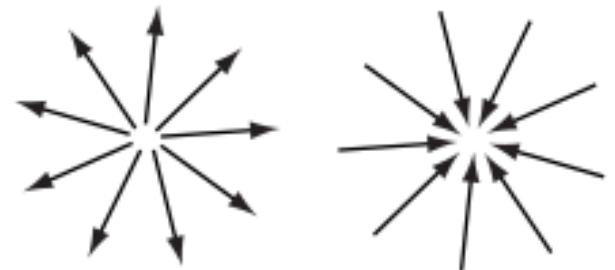
- Given a vector field $\mathbf{v} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, the divergence of $\mathbf{v} = (v_x, v_y, v_z)^T$ is the scalar quantity

$$\operatorname{div} \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}.$$

- Intuitively, if \mathbf{v} is a flow field that transports mass, $\operatorname{div} \mathbf{v}$ characterizes the increase or loss of mass at a given point p in the vector field in unit time.

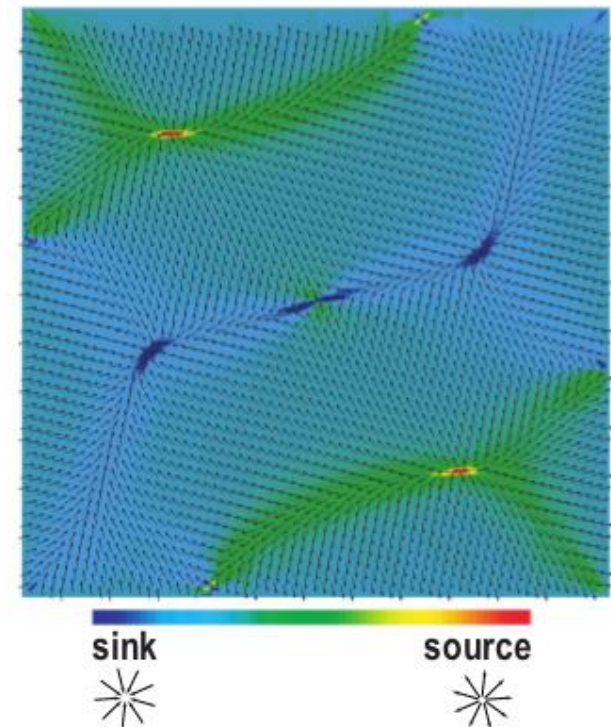
Divergence

- A positive divergence at \mathbf{p} denotes that mass would spread from \mathbf{p} outward. Positive divergence points are called sources.
- A negative divergence at \mathbf{p} denotes that mass gets sucked into \mathbf{p} . Negative divergence points are called sinks.
- A zero divergence at \mathbf{p} denotes that mass is transported without getting spread or sucked, i.e., without compression or expansion.



Divergence

- The example figure shows the divergence of a 2D flow field using a blue-to-red colormap.
- Red areas indicate high positive divergence(sources).
- Blue areas indicate high negative divergence,(sinks).
- We get the image of a flow field that emerges from the sources and ends up in the sinks.



Vorticity



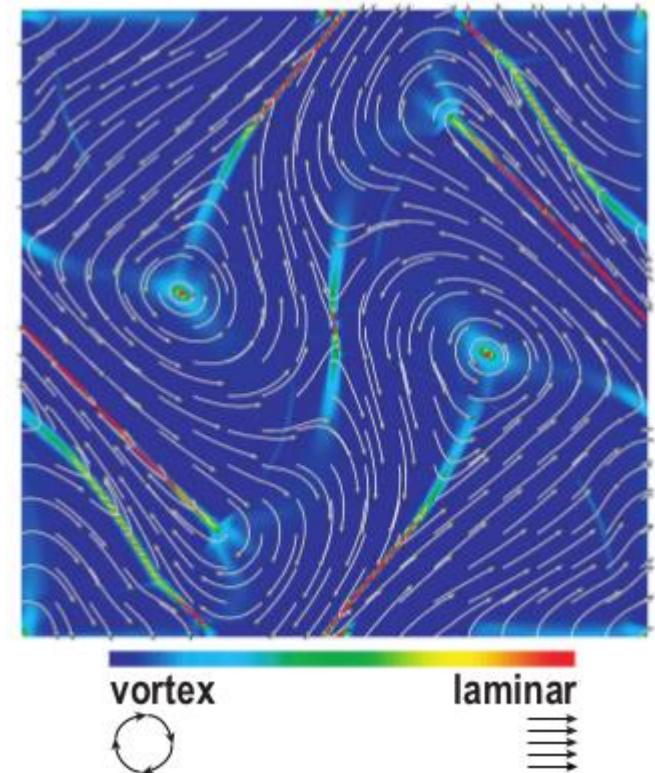
- Given a vector field $\mathbf{v} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, the vorticity of \mathbf{v} , also called the ***curl*** or ***rotor*** of \mathbf{v} , is the vector quantity.

$$\text{rot } \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}, \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}, \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$

- The vorticity ***rot v*** of \mathbf{v} is a vector field that is locally perpendicular to the plane of rotation of \mathbf{v} and whose magnitude expresses the speed of angular rotation of \mathbf{v} around ***rot v***.
- Hence, the vorticity vector characterizes the **speed** and **direction of rotation** of a given vector field at every point.

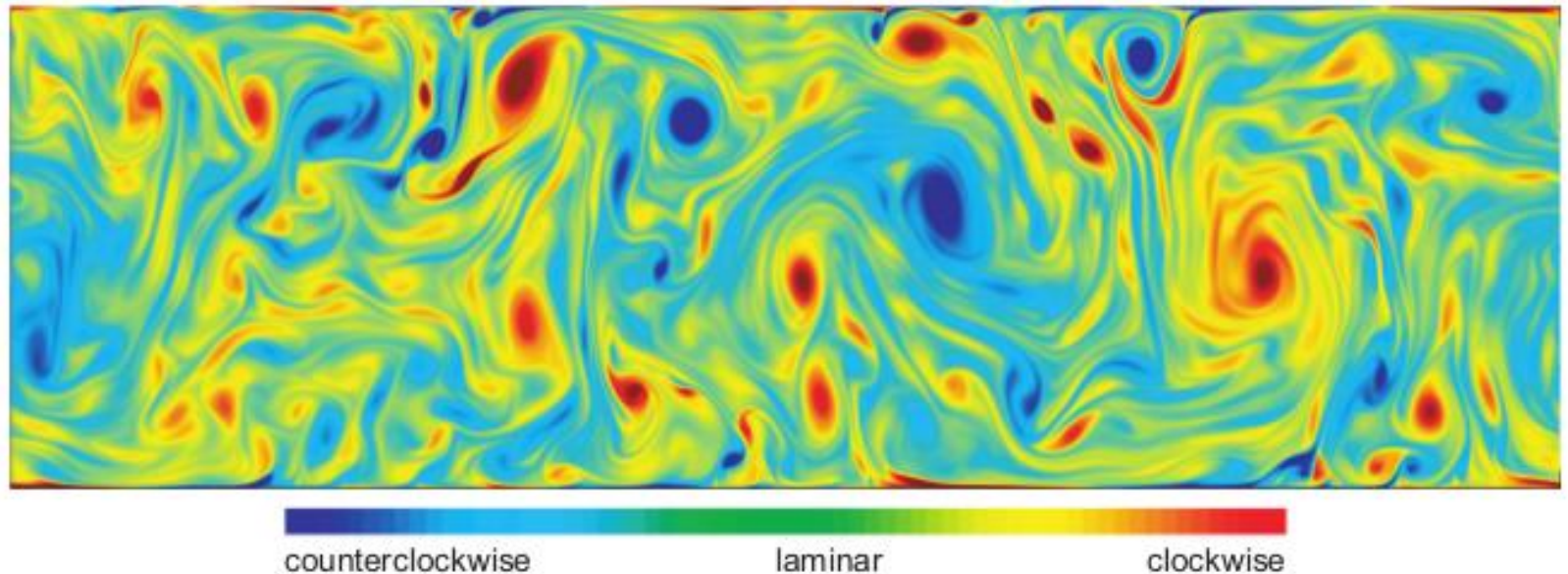
Vorticity

- Blue areas indicate low-vorticity, laminar regions.
- Red areas indicate high-vorticity regions.
- Two small circular red spots indicate localized vortices.
- Several elongated thin red strips indicate areas where the vector field quickly changes direction.



Vorticity

- Visualizes the vorticity of a more complex turbulent 2D flow.
- Blue and red indicate respectively counterclockwise and clockwise spinning vortices.
- Green indicates low-vorticity, laminar regions.
- The image clearly conveys the high complexity of the flow.



Vector Glyphs

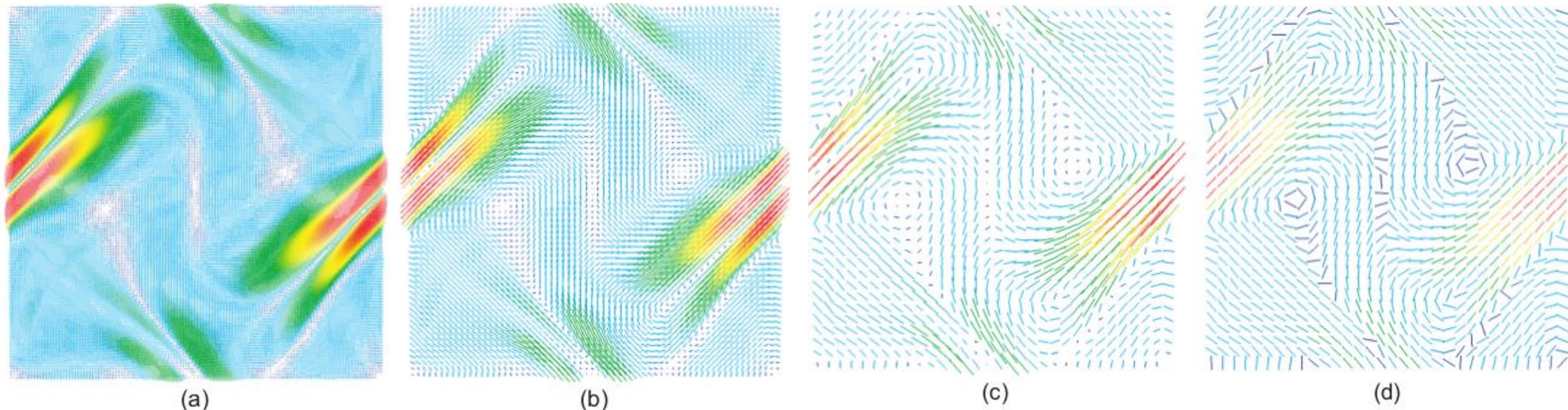
- Vector glyphs are probably the simplest, and fastest, and most popular technique for visualizing vector fields.
- The vector glyph mapping technique essentially associates a vector glyph, or vector icon, with every sample point of the vector dataset.
- Various properties of the icon, such as location, direction, orientation, size, and color, are adjusted to reflect the value of the vector attribute it represents.

Line Glyphs

- Lines essentially show the position, direction, and magnitude of a set of vectors.
- Given a vector dataset defined on a sampled domain D , we associate a line $l = (x, x + kv(x))$ with every sample point $x \in D$ that has a vector attribute $v(x)$.
- The parameter k represents the scaling factor used to map the vector magnitudes to the geometric domain.
- Oriented line glyphs are sometimes also called hedgehogs,

Line Glyphs

- The following figure shows a line glyph, or hedgehog, visualization of a 2D vector field defined on a square domain.



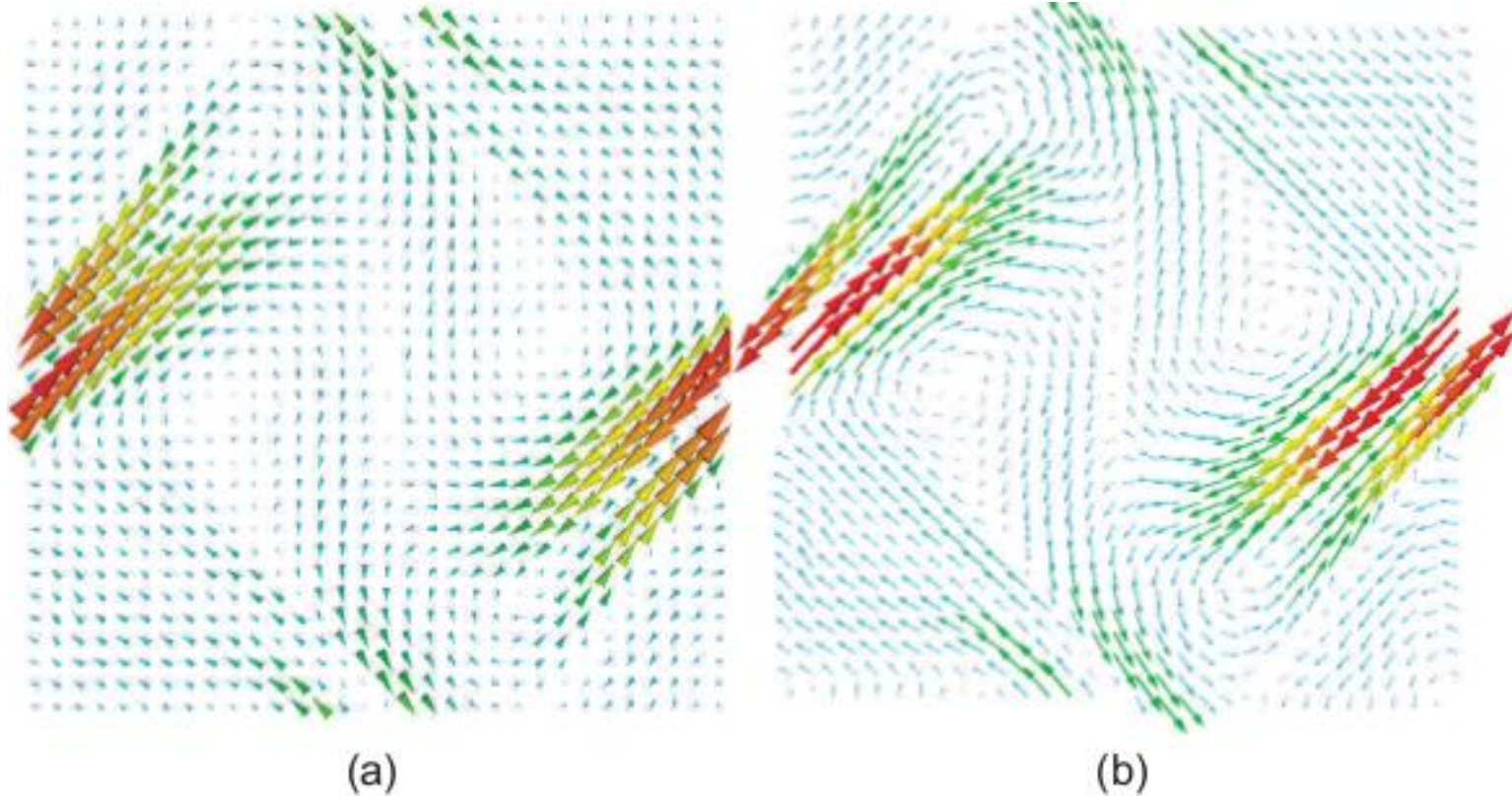
- The clarity of hedgehog visualizations depends strongly on the glyph scaling factor. (a) - rate of 2, (b) - a rate of 4, (c) - rate of 8 and (d) - the vector field is uniformly subsampled at a rate of 8, but the line glyphs are all scaled to the same length.

Cone and arrows



- Cone and arrow glyphs have the advantage of being able to convey a signed direction, whereas lines convey an unsigned direction only.
- Glyphs also take more space to draw, so they increase the clutter or require lower-resolution datasets.
- An interesting compromise between arrows and lines is to use Gouraud shaded lines.
- By shading the line glyph from full color at the glyph origin to the background color at the line tip, a visual effect similar to a thin arrow can be obtained without the need for extra screen space.

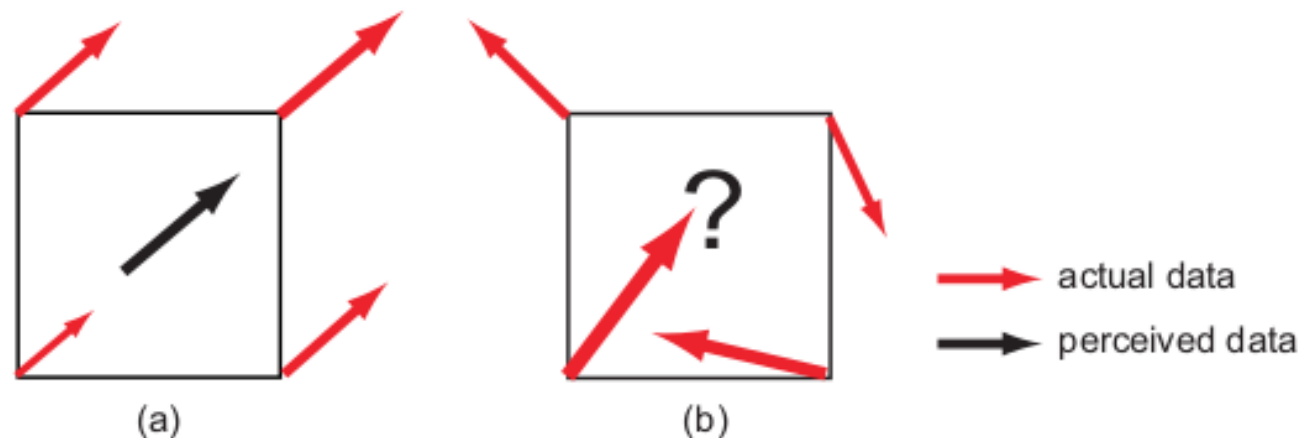
Cone and arrows



The trade-off between the power of expression of glyphs, or number of attributes they can encode, and minimal screen size needed by a glyph is an important characteristic of glyph-based visualizations.

Vector glyphs in 2D

- Consider a zoomed-in detail showing a hedgehog plot over a single cell of a 2D vector field in the figure below.
- In the first case the vector field variation over the displayed cell is quite small.
- There is an increase in magnitude in upper-right direction and orientation.
- In the second case the situation is more problematic.
- Clearly, the interpretation can get very confusing when we have hundreds of cells in this situation.

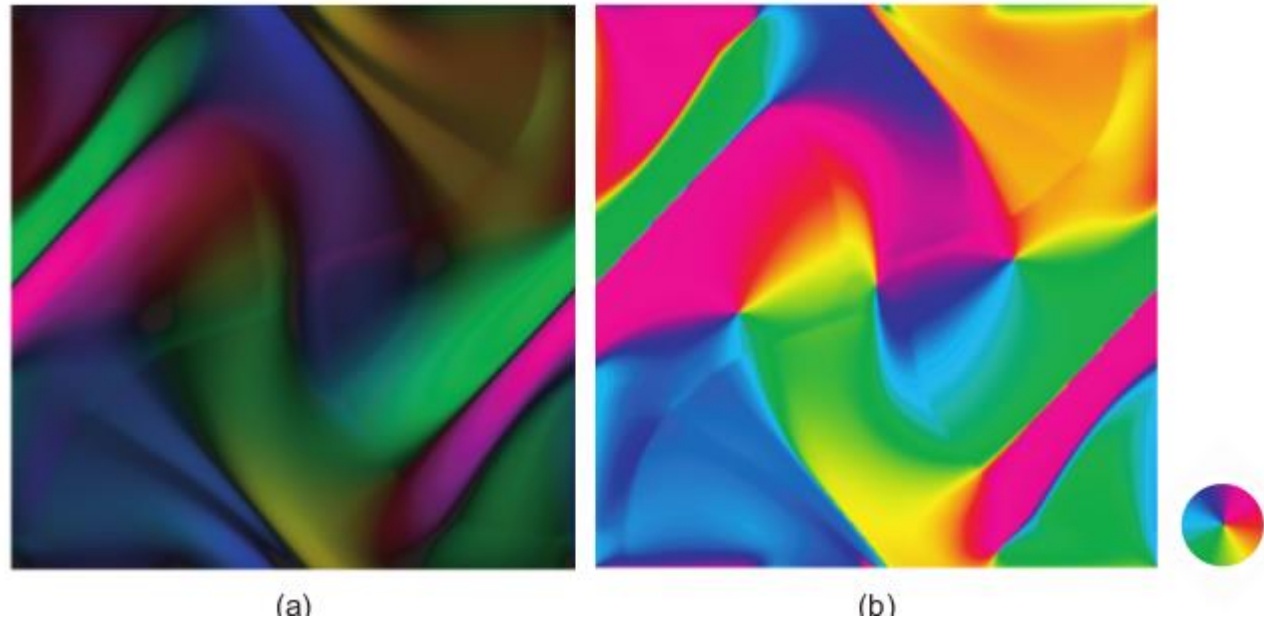


Vector Color Coding

- Color mapped surfaces, have several advantages compared to sparse visualizations, such as glyphs.
- One of the simplest techniques to produce such visualizations is vector color coding.
- **Color coding on 2D surfaces**
 - Low-vector-magnitude regions can be easily detected as dark (low value) areas.
 - high-vector-magnitude regions show up as brightly colored areas.

Color coding on 2D surfaces

- If we are interested only in the vector orientation and not the magnitude, we can set the value component to one, and we obtain the visualization





Thank You